Comments on ‘‘A Reexamination of the Formation of Exhaust Condensation Trails by Jet Aircraft’’

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The jet aircraft condensation trail (contrail) forecast algorithm developed by Hanson and Hanson (1995) (referred to hereafter as HH) suffers from the authors' inadequate treatment of water vapor contributed to contrails by the combustion of jet fuel. The following comments on HH provide additional details to comments furnished by Schumann (1996a). Appleman (1953) accounts for exhaust water vapor contributions by introducing a jet engine factor, or contrail factor, into the development of his contrail forecast algorithm. Appleman defines this contrail factor as the ratio of the water vapor supplied by the combustion of aviation fuel in the jet engine to the increase in the contrail plume temperature caused by heating due to engine exhaust. When the ambient air is initially dry, HH ignore the contribution of water vapor due to the engine exhaust. This can be seen by examining their Eq. (7). This equation (reproduced below) can be solved using an iterative process to find the corresponding critical temperature:

$$\frac{p}{622}\,\mathrm{CF} = \frac{100}{\mathrm{RH}}[10\,\exp(P_1 - P_2 - P_3 - P_4)] \times \ln(10)(P_5 - P_6 + P_7 + P_8), \qquad (1)$$

where p is the atmospheric pressure, CF is the engine contrail engine factor, RH is ambient relative humidity, and P_n are coefficients that are functions of ambient temperature.

Equation (1) has a mathematical singularity at zero relative humidity, but we can rewrite it as follows:

$$\mathrm{RH}\frac{P}{622}\,\mathrm{CF} = 100[10\,\exp(P_1 - P_2 - P_3 - P_4)] \times \ln(10)(P_5 - P_6 + P_7 + P_8). \qquad (2)$$

At 0% ambient relative humidity, this reduces to the expression

$$0 = [10\,\exp(P_1 - P_2 - P_3 - P_4)](P_5 - P_6 + P_7 + P_8).$$

We see that at 0% ambient relative humidity HH's equation for finding the contrail formation critical temperatures reduces to an expression that is independent of the contrail factor CF. Physically, this says that the combustion of jet fuel does not add water vapor to the atmosphere when the air is initially dry, which is obviously not the case. Hanson and Hanson (1995) attempt to justify their approach by asserting that at 0% ambient relative humidity, jet aircraft contrails would occur only when the ambient temperature is ‘‘. . . an extremely large magnitude negative number that would not be characteristic of the physical conditions under examination.’’ The fact that jet aircraft condensation trails are commonly observed in the dry stratosphere—Peters (1993) cites a 61% rate of contrail occurrence at altitudes from 40 000 to 63 000 ft and a 37% rate of occurrence above 63 000 ft in a database of 1040 U-2 aircraft observations—speaks eloquently against HH's approach to the prediction of jet aircraft condensation trails.

The root of HH's difficulty lies in their analysis of the contrail formation process at relative humidities of less than 100%. Their difficulty can be understood by comparing their analysis with Appleman's. Appleman treats contrails as mixing clouds. An isobaric mixing contrail model is illustrated in Fig. 1. Parcels A and B in this figure represent two different ambient atmospheric conditions. When the engine exhaust mixes with parcel A, saturation is reached and a contrail is formed. When the engine exhaust mixes with parcel B, the mixture fails to reach saturation and no contrail forms. The set of temperatures that lies along the line defined by the engine exhaust temperature and vapor pressure tangent to the saturation vapor pressure curve from 0% to 100% relative humidity is the set of critical temperatures in vapor pressure and temperature space. This can be seen by using the same isobaric mixing arguments, as used above for parcels A and B. When the engine exhaust mixes with a parcel lying anywhere on the critical temperature line, the resulting mixture will just reach saturation with respect to water. To make this line op-

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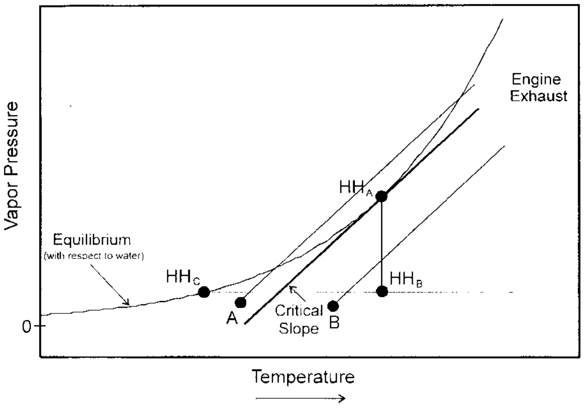

FIG. 1. Examples of parcel mixing in vapor pressure–temperature space.

TABLE 1. Predicted [HH (and Appleman)] critical contrail temperatures (°C).

Pressure (mb)	RH = 1%	RH = 10%	RH = 40%
100	−87.46 (−61.22)	−71.99 (−60.46)	−60.96 (−57.92)
200	−83.14 (−55.14)	−66.66 (−54.32)	−54.87 (−51.60)
300	−80.49 (−51.38)	−63.37 (−50.53)	−51.11 (−47.69)

erationally useful, it must be mapped into atmospheric relative humidity, pressure, and temperature space.

Hanson and Hanson (1995) determine their contrail formation critical temperatures using a two-step process. According to the discussion in the caption of Fig. 1 of HH, the authors begin with a parcel at point HH_A (reference our Fig. 1), defined by the intersection of the ''engine output curve (contrail factor)'' and the saturation vapor pressure curve. In the first step, they find point HH_B by determining the vapor pressure corresponding to the ambient relative humidity. In the second step, they cool the parcel isobarically (''moving horizontally'') until it becomes saturated at point HH_C. They interpret the temperature associated with the point HH_C as the critical temperature for the given ambient relative humidity. The resulting critical temperature line in vapor pressure and temperature space is simply the saturation vapor pressure curve from HH_A at 100% relative humidity. The critical temperatures of HH are unrealistically cold because they fail to correctly account for the moisture added to the contrails by the engine exhaust. It is clear from the graphical description of the HH contrail algorithm why it is impossible for their algorithm to form a contrail in a dry atmosphere; the critical temperature for a dry atmosphere would have to be absolute zero.

The conclusions of HH regarding the improved accuracy of their contrail forecast algorithm warrant further consideration. In Figs. 2 and 3 of HH, the authors use empirical data that originally appeared in Appleman (1957) to compare the performance of their algorithms with Appleman's. The empirical data are the result of Project Cloud Trail, in which pilot observations from fighter interceptor squadrons in the United States were collected in the vicinity of upper-air sounding stations from December 1954 to December 1955. Appleman used the data to estimate the frequency of occurrence of contrails as a function of temperature and pressure only. Appleman states in the first paragraph of section 2.3 of his 1957 report that, ''Assuming the theoretically-derived curves (of minimum relative humidity required for contrail formation) are exact, perfect data would result in the 0% probability curve coinciding with the 100% humidity line, and the 100% probability curve with the 0% humidity line.'' He goes on to point out that, ''Assuming further an equal chance for all relative humidity values at every pressure–temperature point, the 90% and 60% humidity lines should also coincide with the 10% and the 40% probability curves, respectively. However, one would not expect the distribution of mean relative humidity to be constant with altitude; hence, it is only the bounding (theoretical) curves that can be tested.'' It is clear from Appleman's remarks that it is inappropriate for HH to use the Project Cloud Trail empirical data to judge the performance of the HH and Appleman models at relative humidities other than 0% and 100%. In particular, it is inappropriate to use these empirical data to draw the conclusion stated in the caption to Fig. 3 of HH that ''The left-hand area of the graph shows that for an assumed value of the relative humidity of 25%, good agreement between empirical and [HH] theoretical data is obtained.'' The intermediate relative humidity values in HH's Fig. 3 cannot be derived from the flight test data because the flight test only considered the occurrence of contrails as a function of temperature and pressure.

The significance of HH's difficulties can be understood by comparing the critical temperatures T_c predicted by their Eq. (7) with temperatures associated with observed contrails. In the appendix of HH the authors show examples of T_c calculated from both their Eq. (7) and from the Appleman algorithm for a variety of pressure levels and an engine factor of 0.0336 g kg^{-1} K^{-1}. A subset of these data are reproduced in Table 1 below. It is apparent from these data that the T_c predicted by HH at the low relative humidities shown are substantially colder than those predicted by Appleman (1953). An observational dataset became available last fall when the Air Force Phillips Laboratory at Hanscom Air Force Base (AFB), Massachusetts, initiated a multiyear Upper Atmosphere Moisture Field Experiment research program (B. Newton 1996, personal communication). Table 2 shows observed radiosonde temperatures and relative humidities associated with jet aircraft condensation trails collected during the initial phase of this research program on 18 and 19 September 1995 at four sites near Hanscom AFB. It is apparent from a comparison of the

TABLE 2. Observed temperature and relative humidity near contrails.

Pressure (mb)	RH (%)	Temperature (°C)
248.5	10.5*	−49.3
209.3	10.2*	−51.9
248.5	10.4*	−49.3
230.0	10.8*	−51.2
258.6	10.0*	−47.0
230.0	10.5*	−51.2
252.5	7.0**	−46.6
211.5	2.0**	−51.8
254.9	6.0**	−47.3
232.6	5.0**	−50.3
285.9	3.0**	−40.1
213.0	6.0**	−53.2
233.9	5.0**	−48.6
229.6	13.8*	−53.1
209.2	13.0*	−53.4
253.0	12.8*	−48.5

* Measured by VIZ rawinsonde, model 1543.
** Measured by Vaisala rawinsonde, model RS80.

data in Tables 1 and 2 that the critical contrail temperature values predicted by the HH algorithm at low ambient relative humidities are much colder than the observed ambient temperatures associated with contrail occurrence. These data suggest the HH algorithm would significantly underforecast contrails in dry atmospheric conditions.

A direct comparison of the performance of the Appleman and HH algorithms is presented in Table 3 for ambient relative humidities of less than 35%. The ambient pressures, temperatures, and relative humidities shown in Table 3 are from Northrop Grumman Corporation flight test data (Saatzer 1995). The flights employed an instrumented T-33 aircraft to generate contrails and measure ambient conditions (temperature and relative humidity) and aircraft parameters (contrail factor and altitude); a Lear 35 chase aircraft was deployed to observe the resulting contrails. A standard flight profile was to ascend from a no-contrail condition (panel 0) until a contrail just began to form (panel 1), continue to ascend until a strong contrail formed (panel 2), and then descend until the contrail terminated (panel 0). The result of this profile is that the data represent borderline cases. Indeed, if the data were error free, the set of panel 1 calls should lie approximately along the line of critical temperature. The contrail factors measured are in many cases lower than the theoretical minimum (0.028 g kg^{-1} K^{-1}) proposed by Schumann (1996b). Although a complete explanation for this is outside the scope of this note, a possible explanation is that the flight profile in the Northrop Grumman test flights resulted in non-steady-state conditions, similar to those observed in the ‘‘contrails’’ of automobiles in winter. When a cold automobile is started, the contrails are much thicker initially when the engine is cold, and some of the heat produced by combustion warms the engine and exhaust system. Presumably, one would notice a transient decrease in contrail intensity upon throttling back a very hot aircraft engine. It is clear from the data presented in Table 3 that the Appleman algorithm outperformed the HH algorithm for the low relative humidity contrail cases shown.

In the light of these difficulties, it is inappropriate for HH to assert that their algorithm provides improved contrail forecast accuracy, particularly at low ambient relative humidities.

Hanson and Hanson (1995) also contains smaller errors, which should be pointed out. In the first paragraph on page 2402, the authors include three contrail factors for non-, low-, and high-bypass engines, which they attribute to Peters (1993). These factors are not given in Peters (1993), but rather are included in Saatzer (1995). The parameter P_8 is incorrect as printed. To duplicate the HH results, P_8 must be

TABLE 3. Comparison of HH and Appleman contrail forecasts.

P (mb)	RH (%)	Engine factor (g kg K^{-1})	Panel call*	T (°C)	T_c (°C) (Appleman)	$(T - T_c)$** (Appleman)	T_c (°C) (HH)	$(T - T_c)$** (HH)
215.72	28	0.027	1	−53.90	−55.51	1.61	−59.30	5.40
213.89	28	0.028	2	−54.21	−55.26	1.05	−59.06	4.85
206.46	23	0.030	1	−55.50	−55.15	−0.35	−60.46	4.96
210.02	26	0.036	2	−55.00	−53.20	−1.80	−57.67	2.67
212.59	23	0.036	1	−54.30	−53.21	−1.09	−58.64	4.34
216.76	21	0.037	0	−53.65	−52.85	−0.80	−59.02	5.37
272.71	22	0.025	1	−52.34	−54.31	1.97	−60.02	7.68
272.23	22	0.025	0	−52.09	−54.33	2.24	−60.04	7.95
266.61	21	0.026	1	−52.54	−54.20	1.66	−60.28	7.74
269.94	19	0.026	2	−52.15	−54.16	2.01	−61.03	8.88
269.91	21	0.026	1	−52.38	−54.08	1.70	−60.17	7.79
270.12	23	0.025	0	−52.32	−54.36	2.04	−59.72	7.40
276.93	33	0.027	1	−52.28	−53.01	0.73	−55.64	3.36

* Panel call: 0—no contrail, 1—contrail onset, and 2—strong contrail.
** Note that only values of $(T - T_c) < 0$ imply contrails.

$$P_8 = 10.5961 \times 10 \exp\left(3.49149 - \frac{1302.88}{T}\right)\frac{\ln(10)}{T^2}.$$

The author's definitions of e_w and e_s in sections 2 and 3 of HH are inconsistent. In section 2, e_w is defined as the saturation vapor pressure. In the following formulation reproduced from section 3 in HH,

$$\frac{d(e_s)}{dT} = \frac{100}{\mathrm{RH}}\left[\frac{d(e_w)}{dT}\right],$$

it appears that e_w is used to represent the ambient vapor pressure and e_s is used to represent the saturation vapor pressure.

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